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Localization of matter on pure geometrical thick branes

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Abstract: In the literature, several types of thick smooth brane configurations in a pure geometric Weyl integrable 5-dimensional space time have been presented. The Weyl geometry is a non-Riemannian modification of 5-dimensional Kaluza-Klein (KK) theory. All these thick brane solutions preserve 4-dimensional Poincaré invariance, and some of them break Z_2-symmetry along the extra dimension. In this paper, we study localization of various matter fields on these pure geometrical thick branes, which also localize the graviton. We present the shape of the potential of the corresponding Schrödinger problem and obtain the lowest KK mode. It is shown that, for both spin 0 scalars and spin 1 vectors, there exists a continuum gapless spectrum of KK states with $m^2 > 0$. But only the massless mode of scalars is found to be normalizable on the brane. However, for the massless left or right chiral fermion localization, there must be some kind of Yukawa coupling. For a special coupling, there exist a series of discrete massive KK modes with $m^2 > 0$. It is also showed that for a given coupling constant only one of the massless chiral modes is localized on the branes.

Keywords: Large Extra Dimensions, Field Theories in Higher Dimensions.

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1. Introduction

Recently, there has been increasing interest and considerable activity in the study of higher-dimensional space-times with large extra dimensions [1 − 3]. Suggestions that extra dimensions may not be compact [2 − 8] or large [1, 9] can provide new insights for the solution of some relevant problems of high-energy physics such as the mass hierarchy problem, dark matter, non-locality and the cosmological constant [5, 8, 10]. In the framework of brane scenarios, an important ingredient is that gravity is free to propagate in all dimensions, whereas all the matter fields are confined to a 3-brane with no contradiction with present time gravitational experiments [1, 5, 7, 11, 12].

In the brane world scenario, an important question is how to realize the brane world idea, in which a key ingredient is localization of various bulk fields on a brane by a natural mechanism. It is well known that the massless scalar field [13] and the graviton [2] are localized on branes of different types, and that the spin 1 Abelian vector fields can not be localized on the Randall-Sundrum(RS) model in five dimensions but can be localized in some higher-dimensional cases [14]. For fermions, they do not have normalizable zero modes in five and six dimensions [13 − 19]. Meanwhile, for the brane with inclusion of scalar backgrounds [20] and minimal gauged supergravity [21] in higher dimensions, localized chiral fermions can be obtained under some conditions.

Recently, thick brane scenarios based on gravity coupled to scalars have been constructed [22 − 26]. An interesting feature of these models is that one can obtain branes naturally without introducing them by hand in the action of the theory [22]. Furthermore, these scalar fields do not play the role of bulk fields but provide the “material” from which the thick branes are made of. By considering a non-Riemannian modification of 5-dimensional Kaluza-Klein (KK) theory (in a pure geometric Weyl integrable 5-dimensional space time), the generalized models based on gravity coupled to scalars have been studied in refs. [27 − 29]. In this scenario, spacetime structures with pure geometric thick smooth branes separated in the extra dimension arise. The authors obtained a single bound state which represents a stable 4D graviton and proved that the spectrum of massive modes of KK excitations is not discrete or quantized at all, but continuous without mass gap due to the asymptotic behavior of the quantum mechanics potential [28, 29]. This gives an very important conclusion: the claim that Weylian structures mimic classically quantum behavior does not constitute a generic feature of these geometric manifolds [28].

The aim of the present article is to investigate localization of various matters on the pure geometrical thick branes obtained in refs. [27 − 29]. The paper is organized as follows: In section 2, we first give a review of the thick branes arising from a pure geometric Weyl integrable 5-dimensional space time, which is a non-Riemannian modification of 5-dimensional KK theory. Then, in section 3, we study localization of various matters on the pure geometrical thick branes in 5 dimensions. Finally, a brief conclusion and discussion are presented.

2. Review of thick brane worlds arising from pure geometry

Let us start with a non-Riemannian generalization of KK theory, i.e., a pure geometrical Weyl action in five dimensions

$$S_5^W = \int_{M_5^W} \frac{d^5x\sqrt{-g}}{16\pi G_5} e^{\frac{3}{2}\omega} \left[R + 3\tilde{\xi}(\nabla\omega)^2 + 6U(\omega)\right], \tag{2.1}$$

where M_5^W is a 5-dimensional Weyl-integrable manifold specified by the pair (g_{MN}, ω), g_{MN} is a 5-dimensional metric and ω is a Weyl scalar function. In such manifolds the Weylian Ricci tensor is given by $R_{MN} = \Gamma^P_{MN,P} - \Gamma^P_{PM,N} + \Gamma^P_{MN}\Gamma^Q_{PQ} - \Gamma^P_{MQ}\Gamma^Q_{NP}$, with $\Gamma^P_{MN} = \{^{\ P}_{MN}\} - \frac{1}{2}(\omega_{,M}\delta^P_N + \omega_{,N}\delta^P_M - g_{MN}\omega^{,P})$ the affine connections on M_5^W and $\{^{\ P}_{MN}\}$ the Christoffel symbols. The parameter $\tilde{\xi}$ is a coupling constant, and $U(\omega)$ is a self-interaction potential for ω, which, in general, breaks the invariance of the action (2.1) under Weyl rescaling,

$$g_{MN} \to \Omega^{-2} g_{MN}, \quad \omega \to \omega + \ln\Omega^2, \quad \tilde{\xi} \to \tilde{\xi}/(1 + \partial_\omega \ln\Omega^2)^2, \tag{2.2}$$

where Ω^2 is a smooth function on M_5^W. $U(\omega) = \lambda\, e^{\omega}$, where λ is a constant parameter, is the only functional form which preserves the scale invariance of the Weyl action (2.1). When the Weyl invariance is broken, the scalar field transforms from a geometrical object into an observable degree of freedom which generates the smooth thick brane configurations, namely, ω is not a bulk field playing the role of the modulus for the extra dimension. The Weyl action is of pure geometrical nature since the scalar field ω enters in the definition of the affine connections of the Weyl manifold.

The ansatz for the line-element which results in a 4-dimensional Poincaré invariance of the Weyl action (2.1) is given by

$$ds_5^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{2.3}$$

where $e^{2A(y)}$ is the warp factor, and y stands for the extra coordinate.

In search of a solution to the setup defined by (2.1) and (2.3), we shall use the conformal technique. Via a conformal transformation, $\hat{g}_{MN} = e^{\omega} g_{MN}$, we go from the Weyl frame to the Riemann one, $M_5^W \to M_5^R$. The action (2.1) is mapped into the following Riemannian form

$$S_5^R = \int_{M_5^R} \frac{d^5x\sqrt{-\hat{g}}}{16\pi G_5}\left[\hat{R} + 3\xi\left(\hat{\nabla}\omega\right)^2 + 6\hat{U}(\omega)\right], \tag{2.4}$$

where $\xi = \tilde{\xi} - 1$, $\hat{U}(\omega) = e^{-\omega}U(\omega)$. Thus, in this frame, we have a theory which describes 5-dimensional gravity coupled to a scalar field with a self-interaction potential. After this transformation, the line element reads

$$d\hat{s}_5^2 = e^{2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{\omega(y)}dy^2, \tag{2.5}$$

where $2\sigma = 2A + \omega$. If we introduce a new pair of variables $X \equiv \omega'$ and $Y \equiv 2A'$, then the field equations that are derivable from (2.4) with the ansatz (2.5) reduce to the following pair of coupled equations

$$X' + 2YX + \frac{3}{2}X^2 = \frac{1}{\xi}\frac{d\hat{U}}{d\omega}e^{\omega}, \tag{2.6}$$

$$Y' + \frac{3}{2}XY + 2Y^2 = \left(4\hat{U} - \frac{1}{\xi}\frac{d\hat{U}}{d\omega}\right)e^{\omega}. \tag{2.7}$$

As pointed out in [27], this system of equations can be easily solved if one uses the restriction $X = kY$, where k is an arbitrary constant parameter which is not allowed to adopt the value $k = -1$ because the system would be incompatible. This condition leads to a Riemannian potential of the form $\hat{U} = \lambda e^{\frac{4k\xi}{1+k}\omega}$. It turns out that this constrain leads to the following simple brane configurations:

Configuration 1: Z_2-symmetric thick brane. In this case, $-\infty < y < +\infty$ (we recall that, due to orbifold symmetry of the solution, only one half of the extra dimension, say $0 \leq y < +\infty$, is physically relevant). The expressions for the warp factor and the scalar field read [27]

$$e^{2A(y)} = [\cosh(ay)]^b, \quad \omega = kb\ln\cosh(ay). \tag{2.8}$$

where

$$a = \sqrt{\frac{4+3k}{1+k}2\lambda}, \qquad b = \frac{2}{4+3k}, \tag{2.9}$$

and

$$\lambda > 0, \quad k < -4/3. \tag{2.10}$$

Hence, b is negative and the warp factor is concentrated near of the origin $y = 0$. The energy density of the scalar matter is [28]

$$\mu(y) = -\frac{3a^2b}{4\pi G_5}(e^{ay} + e^{-ay})^{b-2}\left[1 + \frac{b}{4}(e^{ay} - e^{-ay})^2\right]. \tag{2.11}$$

This function has two negative minima and a positive maximum at $y = 0$ between them at some $y \neq 0$, and finally it vanishes asymptotically (see figure 1).

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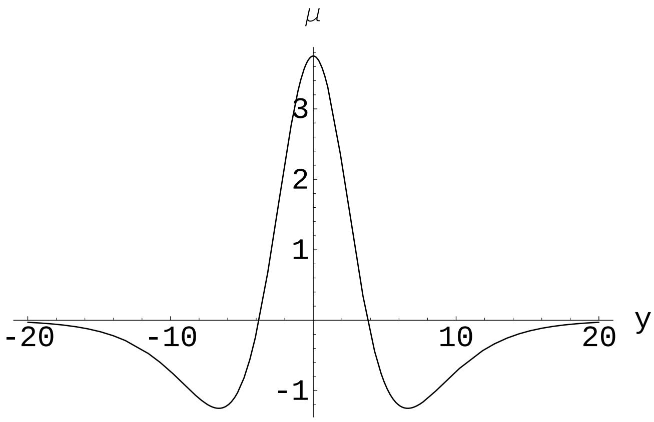

Figure 1: The shape of the energy density function with $k = -5/3$ and $\lambda = 0.01$. A thick brane with positive energy density is centered at the origin $y = 0$.

Configuration 2: non Z_2-symmetric thick brane. The non Z_2-symmetric thick brane solution was found by Barbosa-Cendejas and Herrera-Aguilar [28]

$$e^{2A(y)} = k_3 \left(e^{ay} + k_1 e^{-ay}\right)^b, \quad \omega = \ln\left[k_2 \left(e^{ay} + k_1 e^{-ay}\right)^{kb}\right], \tag{2.12}$$

where k_2 and k_3 are arbitrary constants, and

$$\lambda > 0, \quad k < -4/3, \quad k_1 > 0. \tag{2.13}$$

The Z_2-symmetric solution (2.8) is the particular case of this solution with $k_1 = 1$, $k_2 = 2^{-kb}$ and $k_3 = 2^{-b}$. The parameter k_1 represents the Z_2-asymmetry of the solution through a shift along the extra coordinate. This has a quite important physical implication, i.e., the space time is not restricted to be an orbifold geometry, it allows for a more general type of manifolds.

The 5-dimensional curvature scalar in the Riemann frame and in the Weyl frame are [28]

$$\hat{R}_5 = \frac{-64\lambda k_1(1+k)}{1-k}(e^{ay} + k_1 e^{-ay})^{-(kb+2)}\left[1 + \frac{b(5+3k)}{16k_1}(e^{ay} - k_1 e^{-ay})^2\right] \tag{2.14}$$

and

$$R_5 = \frac{-16a^2 b k_1}{(e^{ay} + k_1 e^{-ay})^2}\left[1 + \frac{5b}{16k_1}(e^{ay} - k_1 e^{-ay})^2\right], \tag{2.15}$$

respectively. The shape of the curvature scalar is plotted in figure 2. It is worth to note that both of them are always bounded. Hence we have a 5-dimensional manifold which is regular in both frames.

Configuration 3: another non Z_2-symmetric thick brane. In above configurations, the parameter ξ has been chosen as $\xi = -(1+k)/(4k)$ with $k \neq -4/3$. In ref. [29], the

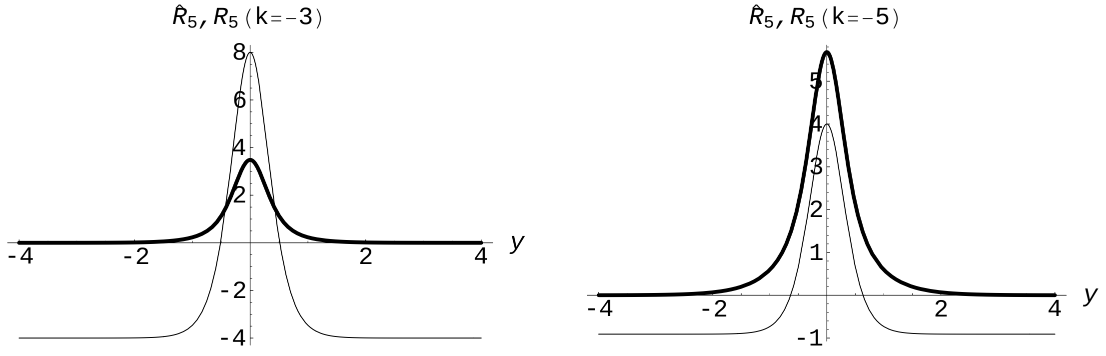

Figure 2: The shape of the curvature scalar $\hat{R}_5$ in the Riemann frame (thick line) and R_5 in the Weyl frame (thin line). The parameters are set to $k_1 = 1$, $\lambda = 1$, $k = -3$ for left panel and $k = -5$ for right panel.

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case $k = -4/3$ is consider and the corresponding solution is read

$$e^{2A} = \left[\frac{\sqrt{-8\lambda p}}{c_1}\cosh\left(c_1(y-c_2)\right)\right]^{\frac{3}{2p}}, \quad \omega = -\frac{2}{p}\ln\left[\frac{\sqrt{-8\lambda p}}{c_1}\cosh\left(c_1(y-c_2)\right)\right], \tag{2.16}$$

where $p = 1 + 16\xi$, c_2 is arbitrary integration constant, and

$$\lambda > 0, \quad p < 0, \quad c_1 > 0. \tag{2.17}$$

From the solution, one can get the energy density of the scalar matter, which behavior is similar to that of (2.11). So, It represents a thick brane with positive energy density centered at $y = c_2$.

These solutions would be utilized to analyze localization of various matter fields on pure geometrical thick branes in the next section.

3. Localization of various matters

Now, we ask the question of whether various bulk fields with spin ranging from 0 to 1 can be localized on thick branes by means of only the gravitational interaction. Of course, we have implicitly assumed that various bulk fields considered below make little contribution to the bulk energy so that the solutions given in previous section remain valid even in the presence of bulk fields. The metric is given by (2.3), but it is more convenient to change it to a conformally flat metric as

$$ds_5^2 = e^{2A}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right), \tag{3.1}$$

in which the relation of the new coordinate z and y is $dz = e^{-A(y)}dy$.

3.1 Spin 0 scalar field

In this subsection we study localization of a real scalar field on pure geometrical thick branes in the backgrounds (2.8)–(2.16). Let us consider the action of a massless real scalar coupled to gravity

$$S_0 = -\frac{1}{2}\int d^5x\sqrt{-g}\, g^{MN}\partial_M\Phi\partial_N\Phi, \tag{3.2}$$

from which the equation of motion can be derived

$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}\partial_N\Phi) = 0. \tag{3.3}$$

By considering the the conformally flat metric (3.1) the equation of motion (3.3) becomes

$$\left(\partial_z^2 + 3(\partial_z A)\partial_z + \eta^{\mu\nu}\partial_\mu\partial_\nu\right)\Phi = 0. \tag{3.4}$$

With the decomposition

$$\Phi(x,z) = \phi(x)\chi(z), \tag{3.5}$$

and demanding $\phi(x)$ satisfies the 4-dimensional massive Klein-Gordon equation $(\eta^{\mu\nu}\partial_\mu\partial_\nu - m^2)\phi(x) = 0$, we obtain the equation for $\chi(z)$

$$\left(\partial_z^2 + 3(\partial_z A)\partial_z + m^2\right)\chi(z) = 0. \tag{3.6}$$

The 5-dimensional action (3.2) reduces to the standard 4-dimensional action for the massive scalars, when integrated over the extra dimension under the conditions that eq. (3.6) is satisfied and the normalization condition

$$\int_{-\infty}^{\infty} dz\; e^{3A}\chi^2(z) = 1 \tag{3.7}$$

is obeyed.

In order to obtain the Schrödinger-like equation, we define $\widetilde{\chi}(z) = e^{\frac{3}{2}A}\chi(z)$ and get

$$\left[-\partial_z^2 + V(z)\right]\widetilde{\chi}(z) = m^2\widetilde{\chi}(z), \tag{3.8}$$

where m is the mass of the KK excitation and the potential is given by

$$V(z) = \frac{3}{2}\partial_z^2 A + \frac{9}{4}(\partial_z A)^2. \tag{3.9}$$

The potential depends only on the warp factor exponent A and has the same form as the case of graviton. For the first and second brane configurations there is a particular case $k = -5/3$ for which one can invert the coordinate transformation $dz = e^{-A(y)}dy$. For the third brane configuration there are two particular cases ($p = -1/4$ and $p = -3/4$). In these cases we can explicitly express y in terms of z. Here we only consider the first brane configuration (2.8), the cases of the other types are similar. For the first case, by taking $k = -5/3$, the expression of y is $y = \mathrm{arcsinh}(az)/a$, and the effective potential is reduced to

$$V(z) = \frac{9\lambda(15\lambda z^2 - 2)}{4(3\lambda z^2 + 1)^2}. \tag{3.10}$$

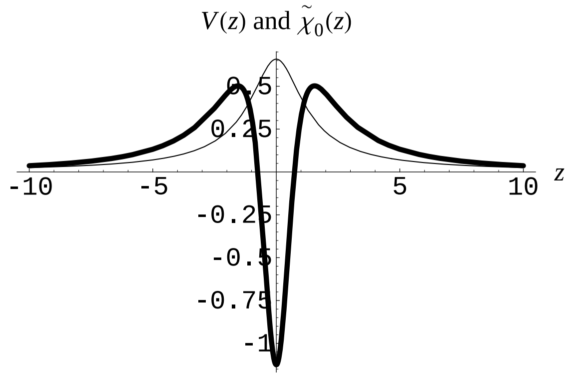

Figure 3: The shape of the effective potential $V(z)$ (thick line) and zero mode $\widetilde{\chi}_0(z)$ (thin line) for scalars. The parameters are set to $k = -5/3$ and $\lambda = 1/4$.

This potential has the asymptotic behavior: $V(z = \pm\infty) = 0$ and $V(z = 0) = -9\lambda/2$. This in fact is a volcano type potential [30, 31]. This means that the potential provides no mass gap to separate the scalar zero mode from KK modes. When $\lambda \to \infty$, this potential tends to the singular one found in the RS scenario [2]. The shape of the above potential is shown in figure 3. For the zero mode $m^2 = 0$, the Schrödinger equation (3.6) can be solved. The only normalizable eigenfunction is turned out to be

$$\widetilde{\chi}_0(z) = \frac{N_1}{(3\lambda z^2 + 1)^{3/4}}, \tag{3.11}$$

where $N_1 = (3\lambda)^{1/4}/\sqrt{2}$ is a normalization constant. This function represents the lowest energy eigenfunction of the Schrödinger equation (3.8) since it has no zeros. In fact, the Schrödinger equation (3.8) can be written as $H\widetilde{\chi} = m^2\widetilde{\chi}$ [22−24], where the Hamiltonian operator is given by $H = Q^\dagger Q$ with $Q = -\partial_z + (3/2)\partial_z\sigma$. Since the operator H is positive definite, there are no normalizable modes with negative m^2, namely, there is no tachyonic scalar mode. Thus the scalar zero mode is the lowest mode in the spectrum. In addition to this massless mode, the potential (3.10) suggest that there exists a continuum gapless spectrum of KK modes with positive $m^2 > 0$, which are similar to those obtained in refs. [3, 22, 23].

3.2 Spin 1 vector field

Let us turn to spin 1 vector field. Here we consider the action of $U(1)$ vector field

$$S_1 = -\frac{1}{4}\int d^5x\sqrt{-g}g^{MN}g^{RS}F_{MR}F_{NS}, \tag{3.12}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ as usual. From this action the equation motion is given by

$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}g^{RS}F_{NS}) = 0. \tag{3.13}$$

From the background geometry (3.1), this equation is reduced to

$$\eta^{\mu\nu}\partial_\mu F_{\nu 4} = 0, \tag{3.14}$$

$$\partial^\mu F_{\mu\nu} + (\partial_z + \partial_z A)\, F_{4\nu} = 0. \tag{3.15}$$

We assume that the A_μ are Z_2-even and that A_4 is Z_2-odd with respect to the extra dimension z, which results in that A_4 has no zero mode in the effective 4D theory. Furthermore, in order to consistent with the gauge invariant equation $\oint dz A_4 = 0$, we use gauge freedom to choose $A_4 = 0$. Under these assumption, the action (3.12) is reduced to

$$S_1 = -\frac{1}{4}\int d^4x dz \left(e^A \eta^{\mu\lambda}\eta^{\nu\rho} F_{\mu\nu}F_{\lambda\rho} - 2\eta^{\mu\nu}A_\mu \partial_z\left(e^A \partial_z A_\nu\right)\right). \tag{3.16}$$

By decomposing the vector field as

$$A_\mu(x,z) = a_\mu(x)\rho(z), \tag{3.17}$$

and importing the normalization condition

$$\int_{-\infty}^{\infty} dz\; e^A \rho^2(z) = 1, \tag{3.18}$$

the action (3.16) is read

$$S_1 = \int d^4x \left(-\frac{1}{4}\eta^{\mu\lambda}\eta^{\nu\rho} f_{\mu\nu}f_{\lambda\rho} - \frac{1}{2}m^2\eta^{\mu\nu}a_\mu a_\nu\right), \tag{3.19}$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ is the 4-dimensional field strength tensor, and we have required that the $\rho(z)$ satisfy the differential equation

$$(\partial_z^2 + (\partial_z A)\partial_z + m^2)\rho(z) = 0. \tag{3.20}$$

For massive vectors, by defining $\widetilde{\rho} = e^{A/2}\rho$, eq. (3.20) changes into

$$\left[-\partial_z^2 + V(z)\right]\widetilde{\rho}(z) = m^2\widetilde{\rho}(z), \tag{3.21}$$

where the potential is given by

$$V(z) = \frac{1}{2}\,\partial_z^2 A + \frac{1}{4}\,(\partial_z A)^2. \tag{3.22}$$

For the first brane configuration, by taking $k = -5/3$, the effective potential is reduced to

$$V(z) = \frac{3\lambda(9\lambda z^2 - 2)}{4(3\lambda z^2 + 1)^2}. \tag{3.23}$$

The potential is very similar to the one given in eq. (3.10). Hence, we encounter the same analyse. The vector zero mode is turned out to be

$$\widetilde{\rho}(z) = \frac{N_2}{(3\lambda z^2 + 1)^{1/4}}, \tag{3.24}$$

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where N_2 is a normalization constant. Now the normalization condition (3.18) is read

$$\int_{-\infty}^{\infty} dz\ \widetilde{\rho}^{\,2}(z) = 1, \tag{3.25}$$

which shows that the vector zero mode is non-normalized. It is turned out that the result is same as the RS model case, i.e. the zero mode of the spin 1 vector field can not be localized on the thick brane. It was shown in the RS model in AdS_5 space that spin 1 vector field is not localized neither on a brane with positive tension nor on a brane with negative tension so the Dvali-Shifman mechanism [32] must be considered for the vector field localization [13].

3.3 Spin 1/2 fermionic field

Localization of fermions in general spacetimes has been studied for example in [20]. In ref. [33], it was found that fermions can escape into the bulk by tunneling, and the rate depends on the parameters of the scalar field potential. In ref. [34], Melfo *et al* studied the localization of fermions on various different scalar thick branes. They showed that only one massless chiral mode is localized in double walls and branes interpolating between different AdS_5 spacetimes whenever the wall thickness is keep finite, while chiral fermionic modes cannot be localized in dS_4 walls embedded in a M_5 spacetime. In this subsection we study localization of a spin 1/2 fermionic field on the pure geometrical thick branes.

Let us consider the Dirac action of a massless spin 1/2 fermion coupled to gravity and scalar

$$S_{1/2} = \int d^5x\sqrt{-g}\left(\bar{\Psi}i\Gamma^M D_M\Psi - \eta\bar{\Psi}F(\omega)\Psi\right), \tag{3.26}$$

from which the equation of motion is given by

$$\left[i\Gamma^M(\partial_M + \omega_M) - \eta F(\omega)\right]\Psi = 0, \tag{3.27}$$

where $\omega_M = \frac{1}{4}\omega_M^{\bar{M}\bar{N}}\Gamma_{\bar{M}}\Gamma_{\bar{N}}$ is the spin connection with $\bar{M}, \bar{N}, \cdots$ denoting the local Lorentz indices, Γ^M and $\Gamma^{\bar{M}}$ are the curved gamma matrices and the flat gamma ones, respectively, and have the relations $\Gamma^M = e^M_{\bar{M}}\Gamma^{\bar{M}} = (e^{-A}\gamma^\mu, -ie^{-A}\gamma^5)$ with $e_M^{\bar{M}}$ being the vielbein. The spin connection $\omega_M^{\bar{M}\bar{N}}$ in the covariant derivative $D_M\Psi = (\partial_M + \frac{1}{4}\omega_M^{\bar{M}\bar{N}}\Gamma_{\bar{M}}\Gamma_{\bar{N}})\Psi$ is defined as

$$\begin{aligned}
\omega_M^{\bar{M}\bar{N}} &= \frac{1}{2}e^{N\bar{M}}(\partial_M e_N^{\bar{N}} - \partial_N e_M^{\bar{N}}) \\
&\quad -\frac{1}{2}e^{N\bar{N}}(\partial_M e_N^{\bar{M}} - \partial_N e_M^{\bar{M}}) \\
&\quad -\frac{1}{2}e^{P\bar{M}}e^{Q\bar{N}}(\partial_P e_{Q\bar{R}} - \partial_Q e_{P\bar{R}})e_M^{\bar{R}}.
\end{aligned} \tag{3.28}$$

The non-vanishing components of ω_M are

$$\omega_\mu = \frac{1}{2}(\partial_z A)\gamma_\mu\gamma_5. \tag{3.29}$$

And the Dirac equation (3.27) then becomes

$$\left\{i\gamma^\mu\partial_\mu + \gamma^5\left(\partial_z + 2\partial_z A\right) - \eta\, e^A F(\omega)\right\}\Psi = 0, \tag{3.30}$$

where $i\gamma^\mu\partial_\mu$ is the Dirac operator on the brane. We are now ready to study the above Dirac equation for 5-dimensional fluctuations, and write it in terms of 4-dimensional effective fields. From the equation of motion (3.30), we will search for the solutions of the chiral decomposition

$$\Psi(x,z) = \psi_L(x)\alpha_L(z) + \psi_R(x)\alpha_R(z), \tag{3.31}$$

where $\psi_L(x)$ and $\psi_R(x)$ are the left-handed and right-handed components of a 4-dimensional Dirac field. Let us assume that $\psi_L(x)$ and $\psi_R(x)$ satisfy the 4-dimensional massive Dirac equations

$$\begin{aligned} i\gamma^\mu\partial_\mu\psi_L(x) &= m\psi_R(x), \\ i\gamma^\mu\partial_\mu\psi_R(x) &= m\psi_L(x). \end{aligned}$$

Then $\alpha_L(z)$ and $\alpha_R(z)$ satisfy the following eigenvalue equations

$$\left\{\partial_z + 2\partial_z A + \eta\, e^A F(\omega)\right\}\alpha_L(z) = \quad m\alpha_R(z), \tag{3.32a}$$
$$\left\{\partial_z + 2\partial_z A - \eta\, e^A F(\omega)\right\}\alpha_R(z) = -m\alpha_L(z). \tag{3.32b}$$

In order to obtain the standard four dimensional action for the massive chiral fermions, we need the following orthonormality conditions

$$\int_{-\infty}^{\infty} e^{4A}\alpha_L\alpha_R dz = \delta_{LR}. \tag{3.33}$$

for α_{L_n} and α_{R_n}.

By defining $\widetilde{\alpha}_L = e^{2A}\alpha_L$, we get the Schrödinger-like equation for the left chiral fermions

$$[-\partial_z^2 + V_L(z)]\widetilde{\alpha}_L = m^2\widetilde{\alpha}_L \tag{3.34}$$

with the effective potential

$$V_L(z) = e^{2A}\eta^2F^2(\omega) - e^A\eta\,\partial_z F(\omega(z)) - (\partial_z A)e^A\eta F(\omega). \tag{3.35}$$

For localization of massive fermions around the brane, the effective potential $V_L(z)$ should have a minimum at the brane. Furthermore, we also demand a symmetry for $V_L(z)$ about the position of the brane. This requires $F(\omega(z))$ to be an odd function of z. So we set $F(\omega(z)) = \partial_z \exp\omega(z)$. Here we only discuss the third configuration of brane (2.16) with $p = -3/4$ and $c_2 = 0$ (for others configurations, the corresponding discuss is similar). Now the potential is reduced to

$$V_L(z) = \frac{64c_1^{14/3}\eta^2z^2}{9\left(c_1^4z^2+6\lambda\right)^{1/3}} - \frac{16c_1^{7/3}\eta\left(c_1^4z^2+9\lambda\right)}{9\left(c_1^4z^2+6\lambda\right)^{7/6}}. \tag{3.36}$$

It is worth noting that the value of the potential at $y = 0$ is given by

$$V_L(0) = -\frac{8}{3}\left(\frac{c_1}{6\lambda}\right)^{1/6} c_1^2\eta. \tag{3.37}$$

For right chiral fermion, the corresponding potential is read

$$V_R(z) = \frac{64 c_1^{14/3}\eta^2 z^2}{9\left(c_1^4 z^2 + 6\lambda\right)^{1/3}} + \frac{16 c_1^{7/3}\eta\left(c_1^4 z^2 + 9\lambda\right)}{9\left(c_1^4 z^2 + 6\lambda\right)^{7/6}}, \tag{3.38}$$

and the value at $y = 0$ is given by

$$V_R(0) = \frac{8}{3}\left(\frac{c_1}{6\lambda}\right)^{1/6} c_1^2\eta. \tag{3.39}$$

The shape of the above effective potentials are shown in figure 4 for different values of η. Both the two potentials have the asymptotic behavior: $V_{L,R}(z = \pm\infty) = \infty$. But for a given coupling constant η, the values of the potentials at $z = 0$ are opposite. For positive η, only the potential for left chiral fermions has a negative value at the location of the brane, which can trap the left chiral fermion zero mode:

$$\widetilde{\alpha}_{L0} \propto \exp\left\{-\frac{8}{5}\eta\left(\frac{\sqrt{c_1^4 z^2 + 6\lambda}}{c_1}\right)^{5/3}\right\}, \qquad (\eta > 0) \tag{3.40}$$

which represents the lowest energy eigenfunction of the Schrödinger equation (3.34) since it has no zeros. In this case, the potential for right chiral fermions is always positive, which shows that there does not exist zero mode. But for the case of negative η, things are opposite and only the right chiral zero mode can be trapped on the brane:

$$\widetilde{\alpha}_{R0} \propto \exp\left\{\frac{8}{5}\eta\left(\frac{\sqrt{c_1^4 z^2 + 6\lambda}}{c_1}\right)^{5/3}\right\}. \qquad (\eta < 0) \tag{3.41}$$

For arbitrary $\eta \neq 0$, both the two potentials suggest that there exists a discrete spectrum of KK modes with positive $m^2 > 0$, which are different form the case of the scalar obtained in the section.

It is worth noting that, in the case of no coupling ($\eta = 0$), both the two potentials for left and right chiral fermions are vanish, and hence there are no any localized fermion KK modes including zero modes.

4. Discussions

In this paper, we have investigated the possibility of localizing various matter fields on pure geometrical thick branes, which also localize the graviton, from the viewpoint of field theory. We first give a brief review of several types of thick smooth brane configurations in a pure geometric Weyl integrable 5-dimensional space time. Some of these thick branes break Z_2-symmetry along the extra dimension.

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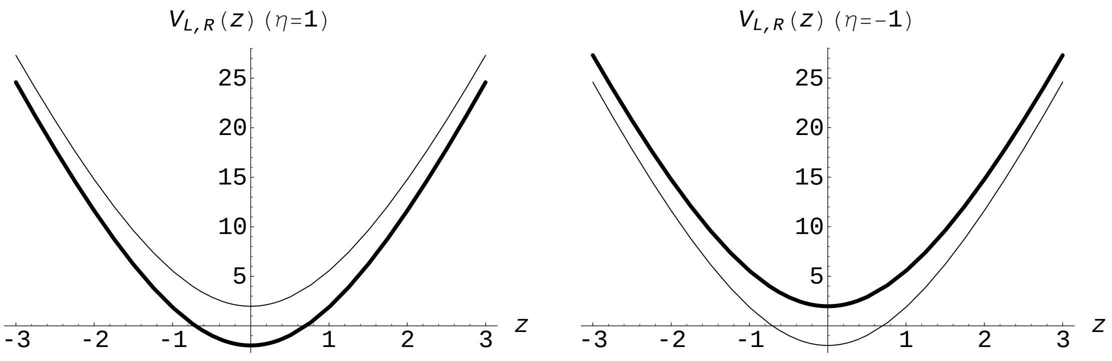

Figure 4: The shape of the potentials V_L and V_R for left and right chiral fermions. The parameters are set to $p = -3/4$, $\lambda = 1$, $c_1 = 1$, and $\eta = 1$ for left panel and $\eta = -1$ for right panel. The thick line stands for the shape of the potentials V_L, and the thin line for V_R.

Then, we check localization of various matter fields on these pure geometrical thick branes from the viewpoint of field theory. For scalars and vectors, the one dimensional Schrödinger potentials are similar to the case of gravity. They have a finite negative well at the location of the brane and a finite positive barrier at each side which vanishes asymptotically. It is shown that there is only a single bound state (zero mode) which is just the lowest energy eigenfunction of the Schrödinger equation for the two kinds of fields. Since all values of $m^2 > 0$ are allowed, there also exist a continuum gapless spectrum of KK states with $m^2 > 0$, which turn asymptotically into continuum plane wave as $|z| \to \infty$ [3, 22, 28, 29]. But the zero mode for spin 1 vector is non-normalized, so vector fields are not localized on the branes. For spin 1/2 fermion, it is shown that, for the case of no Yukawa coupling, there is no bound states for both left and right chiral fermions. Hence, for the massless left or right chiral fermion localization, there must be some kind of Yukawa coupling. These situations can be compared with the case of the domain wall in the RS framework [13], where for localization of spin 1/2 field additional localization method by Jackiw and Rebbi [35] was introduced. In this paper, we consider a special case of coupling as an example. With the special coupling, we get a discrete spectrum of KK modes with positive $m^2 > 0$. However, it is showed that only one massless chiral mode is localized on the branes.

Localizing the fermionic degrees of freedom on branes or defects requires us to introduce other interactions but gravity. Recently, Parameswaran *et al* studied fluctuations about axisymmetric warped brane solutions in 6-dimensional minimal gauged supergravity and proved that, not only gravity, but Standard Model fields could “feel” the extent of large extra dimensions, and still be described by an effective 4-Dimensional theory [21]. Moreover, there are some other backgrounds could be considered besides gauge field [36] and supergravity [37], for example, vortex background [38, 39]. The topological vortex coupled to fermions may result in chiral fermion zero modes [40]. More recently, Volkas *et al* had extensively analyzed localization mechanisms on a domain wall. In particular, in ref. [41], they proposed a well-defined model for localizing the SM, or something close to it,

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on a domain wall brane. Their paper made use of preparatory work done in refs. [42, 43].

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